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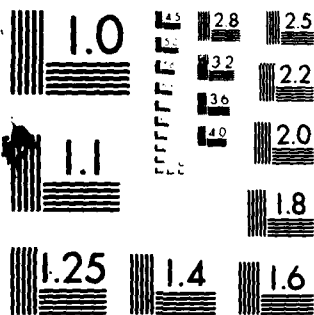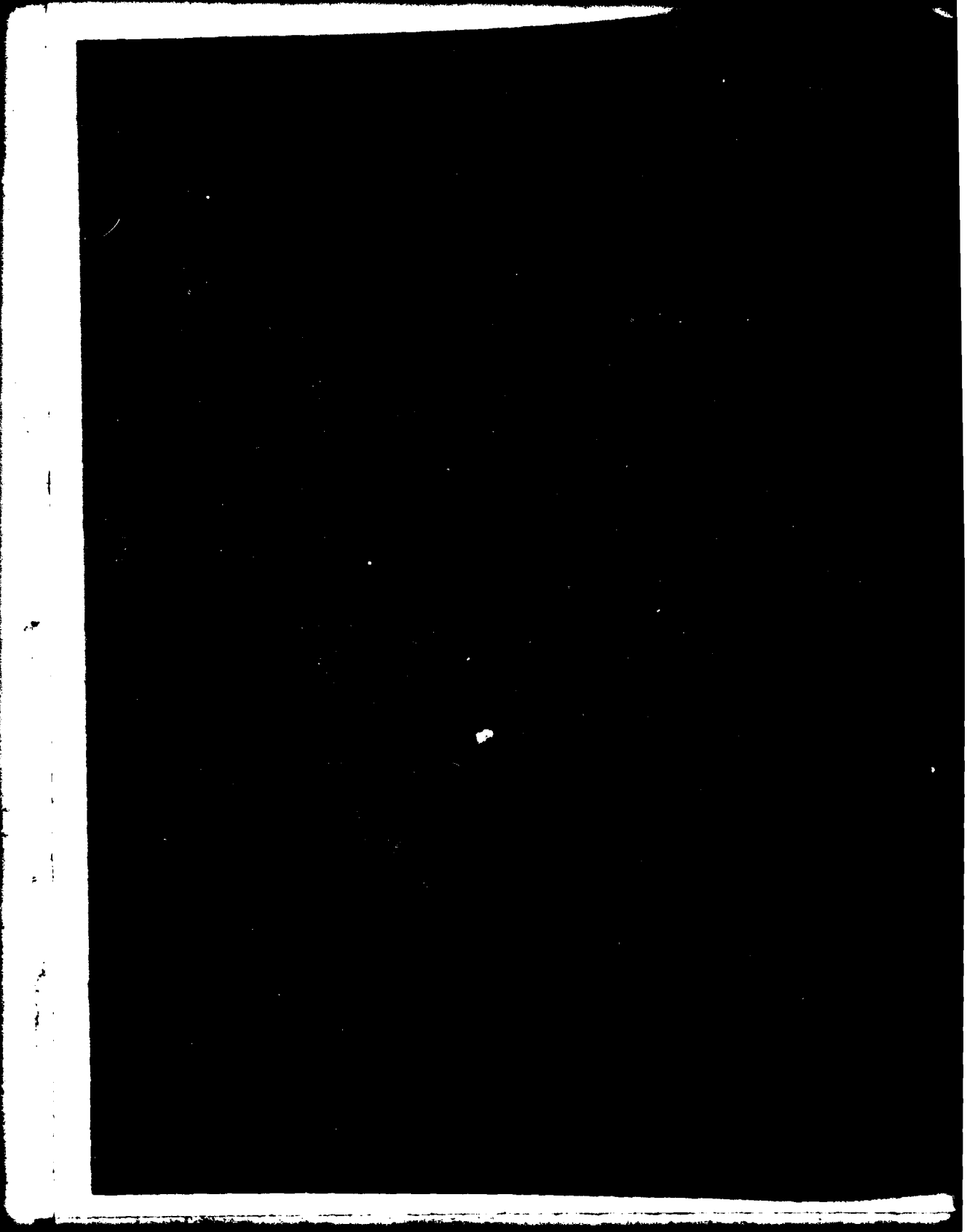UNCLASSIFIED   SCIENTIFIC-1   AFGL-TR-82-0066   NL

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LOW-ORDER MODELS
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Low-order models (LOM's), which are systems of ordinary differential
equations which have been simplified by extreme reduction of the number
of dependent variables, are often capable of representing atmospheric
processes in a qualitatively correct manner. With a LOM it is generally
possible to obtain a much more extended time-dependent solution, or a much
larger ensemble of solutions, than would be economically feasible with
a larger model.

(cont.)

A general procedure for constructing LOM's is described. A selection
of LOM's is presented, to illustrate the many forms which these models
may take and the many uses to which they may be put. The step-by-step
construction of a LOM is illustrated with a model of the large-scale
circulation of a moist atmosphere.

1. Introduction

Some forty years ago, soon after commencing my meteorological education, I reached the conclusion that dynamic meteorology was a subject with many equations and few solutions. In the classroom we devoted much time to the formulation of equations governing the be- havior of the atmosphere. We also studied some special solutions of simplified forms of these equations, such as the familiar "Ekman spiral" solution for the vertical variation of the wind near the earth's surface, but we never considered the nature of the general time dependent solution. It appeared that the principal aim of dynamic meteorology was to produce rational explanations for typical weather phenomena rather than to predict the future evolution of particular weather situations, and we were never taught whether the equations might be used for routine weather forecasting.

It is easy to understand why this situation should have prevailed. The equations possess a form of nonlinearity which makes it unfeasible to determine the general analytic solution. The particular solutions which may be found after suitable simplifications have been introduced are often steady-state solutions, and in any event are rather specialized.

The most prominent nonlinear terms in the equations, and the only ones appearing in some of the popular simplifications, represent the advection of some variable quantity, such as temperature or vorticity, by the wind, which is also a variable quantity. The terms are therefore quadratic, containing products of the advected quantities with the advecting wind. They cannot be removed by any transforma-

tions of the independent or dependent variables.

In the 1940's and earlier, the standard prodecure for obtaining approximate time-dependent solutions of the equations was linearization. This involves first finding a particular exact solution; very often this represents a steady state. Small departures from this solution are then governed approximately by a system of homogeneous linear equations. If the original solution is simple enough, the coefficients in the linear system reduce to constants, and the solutions are exponential or trigonometric functions of time. It should be noted that linearization does not remove the effects of advection; it simply replaces the product of two unknown quantities by the product of an unknown and a known.

The most justifiable use of linearization is the investigation of the stability of the original solution with respect to small perturbations. However, since linearization formerly afforded the only method for finding approximate time-dependent solutions, these were often considered acceptable even when their departures from the original solution could not logically be called "small".

The appearance of computers a few years later changed the situation completely. It soon became obvious that computers could obtain approximate solutions to the meteorological equations by stepwise numerical integration. The most spectacular advance was "numerical weather prediction" - the production of short-range forecasts by obtaining time-dependent solutions originating from observed initial conditions. This development was soon followed by numerical simulation

of the general circulation, which is like numerical prediction except that the initial conditions need not be drawn from real weather situations, and the integrations are extended for weeks or longer rather than days. The purpose of numerical simulation, however, is the explanation of observed features rather than the production of forecasts, and in this respect it is more in keeping with the original aims of dynamic meteorology.

The advent of computers not only changed our procedures for solving the equations; it also changed our way of thinking about them. The analytic function of time had been the only mathematically "true" solution; it incidentally allowed one to express a final state directly in terms of an initial one. The numerical solution, where one advanced through a sequence of "irrelevant" intermediate states to obtain a final one, had been a curious and wasteful approximation; suddenly it became the natural solution, and, in some minds, the "true" one.

When computers were new in meteorology, they were expensive, and unavailable to the majority of dynamic meteorologists. For those who did have access, solution of an equation by computer had to be preceded by many days of preparation. It presently occurred to me that if the equations were sufficiently simplified, perhaps to the point where they could not produce good weather forecasts, but where they still might qualitatively reproduce some features of the general circulation, they might in the same number of days, or perhaps considerably fewer, be solved by slide-rule or hand computation, using

the newly acceptable numerical procedures. This proved to be the case; the highly simplified systems have since become known as low-order models (LOM's) - a term apparently introduced by Platzman (1960).·

It might seem that LOM's would have proven to be a temporary measure, to be abandoned once computers became more powerful and more readily available. This was not the case. Whenever computers have become big enough to do, with detailed systems of equations, what they could previously do only with simple systems, there have always been still bigger jobs to be done. Moreover LOM's, even if originally conceived as a means of bypassing the need for computers, are ideally suited for computer solution. Thus, when computers were finally able to make a 24-hour forecast in a few minutes with a detailed model, they could simulate many years of weather in the same time with a LOM. When they could economically simulate many years of data with a detailed model, they could produce a large ensemble of many-year-long solutions with a LOM.

In this review I shall present a selection of low-order models, chosen to illustrate the wide variety of ways in which they may be formulated, and the wealth of uses to which they may be put. To a considerable extent I shall be giving an account of my own experience with LOM's, from one conceived a quarter century ago, and solved by hand computation, to one still being developed, and requiring a moderatley powerful computer. However, I shall include a fair number of models developed by other investigators, in order that this review may better serve its intended purpose. Had I written this article

some fifteen years ago, I might have attempted an exhaustive survey, but today it is altogether impractical to describe all LOM's which have appeared in the meteorological literature, or even all of the important ones; there are too many.

2. Construction of a low-order model

In theoretical studies of the atmosphere the governing equations, if they enter explicitly, are invariably simplified in various ways before any attempt is made to solve them. The most nearly exact equations which we can formulate are far too complicated. The choice of simplifications is dictated by the particular problem to which the equations are to be applied.

For example, when we are interested only in the larger scales of motion, we ordinarily omit the description of the superposed smaller scales, and introduce the combined effect of the smaller scales upon the larger scales into the equations in terms of exchange coefficients. In addition we usually replace the vertical equation of motion by the hydrostatic equation. If instead we are interested only in the smaller scales, we generally specify in advance the larger-scale field on which they are superposed.

In definitive studies of tropical circulations we must recognize water vapor and liquid water as atmospheric constituents, and evaporation from the ocean and land as a fundamental process, but, when we are interested only in the higher latitudes, we often omit water altogether, and treat the atmosphere as an ideal gas. In addition,

after expressing the horizontal velocity in terms of its vorticity and divergence, we often replace the divergence equation by a quasi-geostrophic approximation.

Without a good description of the water vapor and clouds we cannot accurately specify the incoming and outgoing solar and terrestrial radiation, and we sometimes replace this by Newtonian cooling. In other instances we omit thermal forcing and thermal and mechanical damping altogether. Additional simplifications in common use are the omission of mountains and smaller orographic features, the replacement of the ocean and land areas by a homogeneous underlying surface, and the replacement of this spherical underlying surface by an infinite or bounded plane.

If we are now to solve the equations by computer, we must make further modifications. We must somehow represent the field of each of the N dependent variables (wind components, temperature, etc.) by a finite set of numbers. Usually we first introduce a set of L horizontal levels or layers, and replace each three-dimensional field by L two-dimensional horizontal fields, one for each level. Vertical derivatives or integrals are replaced by finite differences or sums. There are variants where different levels or different values of L are used for different dependent variables.

Next, we introduce a set of M grid points in each horizontal layer, and replace each horizontal field by M numbers, one for each grid point. Horizontal derivatives are replaced by finite differences. There are variants where different grid points are used for different horizontal fields.

An alternative procedure which is being used more and more frequently is to introduce a set of M spatially orthogonal functions, and approximate each horizontal field by a linear combination of these functions. The coefficients in the linear combinations become the new dependent variables. Horizontal derivatives are expressed as linear combinations of horizontal derivatives of the orthogonal functions, which in turn are approximated by linear combinations of the orthogonal functions. There are variants where different sets of orthogonal functions are used for different horizontal fields.

Finally, we use some scheme to replace the time derivatives by time differences. We can then solve the resulting system of LMN difference equations by stepwise numerical integration.

The construction of a low-order model is the same as that of a more general model, except that L and M, and often N, are chosen to be very small. The minimum allowable values depend upon the phenomena being investigated. For large-scale motion, L = 1 may reveal the barotropic processes, while L = 2 may capture the principal baroclinic processes. Grid points tend to be unsuitable for LOM's, since finite differences are unlikely to afford good approximations to horizontal derivatives when M is too small. Most LOM's have therefore been based on orthogonal functions. Some of the nonlinear interactions which render the original equations intractable may be captured when M = 3.

In detail, a LOM may be developed as follows. Let the dependent variables in the horizontally continuous equations, after the desired

physical simplifications have been introduced, and after the vertical

continuum has been replaced by L levels, be $X_1,..,X_K$, where $K = LN$,

and let the equations be

$$\partial X_i / \partial t = \sum_{j,k} A_{ijk}(X_j, X_k) + \sum_j B_{ij}(X_j) + C_i, \quad (2.1)$$

where t is time, A_{ijk} is a quadratic operator which is linear in

X_j and also in X_k, B_{ij} is a linear operator, and C_i is independent

of $X_1,...$ Eq. (2.1) is not completely general, since in the real

atmosphere there are important nonlinear processes, such as radiation

and condensation, which are not quadratic, but it includes many

familiar non-operational atmospheric models.

Corresponding to each variable X_i, choose a set of M_i orthogonal

functions ϕ_{im}, for $m = 0,..,M_i-1$, satisfying any boundary conditions

satisfied by X_i, and satisfying the relations

$$\overline{\phi_{im} \phi_{in}} = \delta_{mn} , \quad (2.2)$$

where the bar denotes an average over the horizontal region in which

X_i is defined. We may then approximate X_i by

$$X_i = \sum_{m=0}^{M_i-1} X_{im} \phi_{im} , \quad (2.3)$$

where

$$X_{im} = \overline{X_i \phi_{im}} . \quad (2.4)$$

It is often convenient to choose $\phi_{i0} = 1$. In many cases the same set of functions ϕ_{im} is chosen for each variable X_i. Frequently the chosen functions ϕ_{im} are eigenvalues of some equation; for example, we may require that

$$\nabla^2 \phi_{im} = -\lambda_{im} \phi_{im} \quad , \qquad (2.5)$$

where ∇^2 is the horizontal Laplacian operator. Trigonometric functions satisfy (2.5).

Upon substituting (2.3) with suitable indices into the right side of (2.1), multiplying by ϕ_{im}, and averaging, we obtain the system

$$\frac{\partial X_{im}}{\partial t} = \sum_{j,p,h,q} a_{imjphq} X_{jp} X_{hq} + \sum_{j,p} b_{imjp} X_{jp} + c_{im} \quad , \qquad (2.6)$$

where

$$a_{imjphq} = \overline{\phi_{im} A_{ijh}(\phi_{jp}, \phi_{hq})} \quad , \qquad (2.7)$$

$$b_{imjp} = \overline{\phi_{im} B_{ij}(\phi_{jp})} \quad , \qquad (2.8)$$

$$c_{im} = \overline{\phi_{im} C_i} \quad . \qquad (2.9)$$

If K and M_i are sufficiently small, Eq. (2.6) defines a low-order model.

The derivation of equations like (2.6) from equations like (2.1) is straightforward. What is not so straightforward is the original choice of physical simplifications, and the subsequent selection of the set of orthogonal functions, or grid points. Ideally these should be guided by the particular phenomena in which the modeler is interested, and the specific questions which he hopes to answer.

For example, if we are simply interested in some of the properties of geostrophic motion, a quasi-geostrophic model, in which the stream-function and isobaric-height fields are identified with each other, may be appropriate. If, on the other hand, we wish to explain _why_ the motion tends to be geostrophic outside of the tropical regions, we need a model whose physical formulation allows the atmosphere to choose between geostrophic and ageostrophic motion. A primitive-equation model is then called for.

Guidelines of this sort have not always been followed. In fact, many LOM's have been constructed by modelers who were not seeking answers to any preformulated questions at all. Such models may nevertheless serve a useful purpose; for example, they may illustrate some atmospheric phenomenon in a more comprehensible manner than would be possible with a less simplified model. Moreover, it sometimes happens that a LOM provides answers to questions formulated after the fact. Finally, a LOM constructed by one investigator is often found by another investigator to possess new potential uses.

3. Some models of conservative systems

Although the circulation of the atmosphere owes its existence
to thermal forcing, and is tempered by thermal and mechanical dissi-
pation, much work in dynamic meteorology has been based on systems
of equations which have been simplified by omitting forcing and
damping altogether. Such systems of equations are often called
conservative, since they effectively assume that the total energy
of the atmosphere does not change. Among conservative systems
were the first models used for numerical weather prediction. The
rationale was that regardless of how important past forcing and
damping may have been in bringing about the weather situation at
forecast time, the effect of the additional forcing and damping
during the next 24 hours or so should be minor. Like the earliest
numerical-weather-prediction models, the first LOM's were conservative.

Probably most meteorologists would agree that the simplest
system of nonlinear partial differential equations derivable from
the atmospheric equations by physical simplifications consists of
a single equation, the barotropic vorticity equation

$$\partial \nabla^2 \psi / \partial t = -J(\psi, \nabla^2 \psi + f) \quad , \qquad (3.1)$$

where ψ is a stream function for the horizontal velocity, f is the
Coriolis parameter, and J is a Jacobian with respect to
horizontal variables. In the simplest variant of (3.1), f is
treated as a constant and its gradient vanishes.

To derive (3.1) from the atmospheric equations we treat the
atmosphere as a homogeneous fluid, neglect all irregularities of
the earth's surface, omit all forcing and damping, and suppress all
vertical variations of the horizontal velocity field. We may take
the earth's surface to be spherical, but it is simpler to treat it
as an infinite or bounded plane. Eq. (3.1) is a statement of the
conservation, at a point moving with the flow, of absolute vorticity,
or, with f constant, simple vorticity, and as such it arises in
a wide variety of fluid dynamical problems. It is distinguished
by possessing two quadratic invariants – the kinetic energy and
the enstrophy. It has formed the basis for many meteorological
studies where simplicity has been deemed important, and it is a
natural starting point for the development of a LOM.

Eq. (3.1) is a special case of (2.1). Since $K = 1$, the sub-
scripts i, j, and k are superfluous, and we shall omit them, but,
for convenience, we shall replace the subscripts m, p, and q by
double subscripts. If we define ψ over an infinite plane, and re-
quire that ψ vary periodically in both the eastward and northward
directions, suitable orthogonal functions satisfying (2.5) as well
as (2.2) are the trigonometric functions

$$\phi_{mn} = e^{i(max + nby)} , \qquad (3.2)$$

where m and n are integers, a and b are real constants, x and y are
eastward and northward distances, and i is now the imaginary unit
rather than an index. Eq. (2.3) becomes

$$\psi = \sum_{m,n} \psi_{mn} \phi_{mn} \; , \tag{3.3}$$

where ψ_{mn} may be complex, and ψ_{mn} and $\psi_{-m,-n}$ must be conjugates to make ψ real.

In constructing what to our knowledge was the first nonlinear meteorological LOM (see Lorenz 1960), we restricted the values of m and n to -1, 0, and 1, making M = 9. We noted further that ψ_{00} is superfluous, while if ψ_{01}, ψ_{10}, ψ_{11}, and $\psi_{1,-1}$ are initially real, they remain real, and if $\psi_{11} = -\psi_{1,-1}$ initially, $\psi_{11} = -\psi_{1,-1}$ always, so that effectively M reduces to 3. We may then let

$$\psi = A \cos ax + B \cos by + C \sin ax \sin by \; , \tag{3.4}$$

and write the equations of the LOM as

$$dA/dt = \alpha\, BC \; , \tag{3.5}$$

$$dB/dt = \beta\, CA \; , \tag{3.6}$$

$$dC/dt = \gamma\, AB \; , \tag{3.7}$$

where the values of the constants α, β, and γ depend upon a and b, and satisfy the relations

$$a^2 \alpha + b^2 \beta + \frac{1}{2}(a^2 + b^2) \gamma = 0 \quad , \qquad (3.8)$$

$$a^4 \alpha + b^4 \beta + \frac{1}{2}(a^2 + b^2)^2 \gamma = 0 \quad . \qquad (3.9)$$

Physically B represents the strength of westerly and easterly currents at alternating latitudes, and A and C together define the amplitude and phase of superposed waves.

Eqs. (3.5)-(3.7) possess several interesting properties. First, the total kinetic energy E and the enstrophy V, where

$$4E = a^2 A^2 + b^2 B^2 + \frac{1}{2}(a^2 + b^2) C^2 \quad , \qquad (3.10)$$

$$4V = a^4 A^2 + b^4 B^2 + \frac{1}{2}(a^2 + b^2)^2 C^2 \quad , \qquad (3.11)$$

are quadratic invariants. More generally, it may be shown that an invariant of any system of partial differential equations will become an invariant of a spectrally derived LOM, provided that the equations and also the invariant are quadratic. This result greatly increases the potential usefulness of LOM's in general.

An immediate consequence of the invariance of E and V is that (3.5)-(3.7) are easily solved analytically. The solutions are elliptic functions of time, and hence periodic, the particular elliptic functions corresponding to A, B, and C depending upon the ratio b/a and the initial conditions. If b = a, C becomes constant, and the elliptic functions degenerate to circular functions.

The LOM defined by (3.5)-(3.7) was derived not to solve a particular problem, but to demonstrate that LOM's could be constructed in a rational manner, and to illustrate various atmospheric phenomena which are intrinsically nonlinear. We describe two of these.

First, when the large-scale waves, represented by A and C, are superposed on a stable zonal current, represented by B, they alter their shape periodically, so as to produce alternate convergences and divergences of momentum flux into and out of the latitudes of the strongest westerlies, and the westerlies respond by alternately increasing and decreasing in strength. This decidedly nonlinear phenomenon is also a property of Eq. (3.1), but it cannot be illustrated by solutions of (3.1) which can be determined analytically. Second, if small-amplitude waves are superposed on an unstable zonal current, they will amplify, in agreement with linear theory, but in drawing their energy from the zonal current they will alter it to the point where it no longer supports further growth. This process is well represented by the elliptic-function solutions.

The model is somewhat restrictive in that (3.4) holds the cyclonic and anticyclonic centers at specific latitudes and longitudes, while, at intermediate latitudes, (3.5)-(3.7) do not permit the longitudes of the troughs and ridges to vary independently of their amplitudes. Platzman (1960) noted that if, either on the plane or the sphere, this restriction is removed by using four real orthogonal functions instead of two to represent the waves, the solutions will still consist of elliptic functions. Subsequently Platzman (1962) made a

systematic study of all LOM's derivable from Eq. (3.1) whose exact
solutions could be determined analytically. He found that the LOM's
could contain as many as three interacting complex orthogonal functions,
and hence up to six real variables. In all cases the solutions were
elliptic functions, except when they degenerated to circular func-
tions or constants.

Conservative LOM's proved to have many uses besides those which
were originally visualized. We mention first a work of Epstein (1969)
in which he introduced the idea of stochastic dynamic prediction.
This involved deriving equations governing the evolution of
ensemble means and covariances rather than individual
states. The rationale was that since the initial state was in any
case uncertain, an ensemble-mean state might have the greatest chance
of being a good forecast, while the variance would provide a measure
of the confidence to be placed in the forecast.

As a system of equations to which first to apply his method,
Epstein chose the 3-variable LOM given by (3.5)-(3.7). He extended
the integration for 10 days, and found, among other things, that the
variations of the ensemble mean did not follow those of any particular
member of the ensemble.

Motivated by the same considerations, but using a different
approach, Paegle and Rohl (1977) followed the evolution of an ensemble
mean of solutions of (3.5)-(3.7) by computing individual time-
dependent solutions and then averaging them. Even though each indivi-
dual solution was periodic, the periods of different solutions were

different, and the ensemble mean decayed toward the long-term mean as the range of the integrations increased.

A considerably different use for (3.5)-(3.7) was found by Lilly (1965), who modified the system by adding a fourth orthogonal function proportional to cos ax cos by. He then used the model, whose exact elliptic-function solution was known, to test a number of frequently used time-differencing schemes.

Conservative LOM's need not be restricted to barotropic flow. Sasaki (1967) formulated a model of thermal convection, which is almost unique among LOM's in that it uses grid points instead of orthogonal functions. Sasaki considered a system which was invariant in one horizontal direction, and, in a vertical cross section, he chose a grid of eight points, defining the temperature at four of these, and the horizontal and vertical velocity components each at two. His resulting model possessed several invariants, including total energy. He obtained analytic solutions, which, not surprisingly, were elliptic functions. Like the earlier conservative LOM's, his model was designed more for illustration than explanation. His solutions demonstrated the growth of convective motions superposed on an unstably stratified state, in agreement with linear theory, but they also illustrated the deceleration and eventual cessation of the growth.

4. Some models of forced dissipative systems

An advantage of LOM's which was particularly important when

computers were slower is the relatively little labor needed to
generate extended solutions, from which long-term means and other
climatological statistics may be evaluated. Conservative models,
which are quite appropriate for some short-term problems, often pro-
duce ludicrous weather patterns when integrated for too long, and in
any case the statistics which they generate are highly dependent
upon the chosen initial state. Thus there are occasions when LOM's
of forced dissipative systems are particularly appropriate.

Unlike the first conservative LOM's, the first forced dissipative
LOM's were conceived in order to answer a rather specific question.
This concerned the potential value of certain empirical weather-fore-
casting procedures. By the middle 1950's numerical weather prediction
was becoming an established discipline, and was gaining an in-
creasing number of devotees. At the same time a smaller but equally
devoted group was favoring statistical weather prediction, based on
empirically derived formulas. The most easily established empirical
formulas are linear, and, among some of the latter group, the idea
became established that the performance of any non-linear formula
could be duplicated by a linear formula, if the latter contained as
predictors both present and past values of the quantities appearing
in the former. As one who had devoted some effort to both numerical
and statistical forecasting, I doubted that this idea was right, and
I proposed to test it by taking a system of nonlinear differential
equations and solving it numerically. The time-dependent solution
would then be treated as data, and would be used to establish a linear

empirical prediction formula by standard procedures. Perfect non-linear prediction of the data could be realized simply by solving the equations again, and I doubted that the linear formulas would even be nearly perfect.

It appeared that while any system of nonlinear equations might suffice for the test, some useful by-products might result from choosing a system resembling the atmospheric equations. Since the equations used in numerical weather prediction were far too compli-cated to allow sufficiently extended integrations to be performed by the computers of that day, I proposed using a LOM.

A model of thermally forced and thermally and mechanically damped baroclinic flow was subsequently constructed and made to run by Bryan (1959). The model used spherical geometry, and its 14 dependent variables were the coefficients of six spherical-harmonic functions in each of two layers, plus a variable mean temperature and static stability. The model demonstrated that models could be made to run forever; it was the first meteorological model to simu-late a full year of data. It also afforded a fair qualitative repre-sentation of some of the principal features of the general circulation.

The model lacked one feature needed for the proposed test; its solution was too regular, and prediction of its future behavior was a trivial matter. I subsequently sought a model which would be no more complicated than Bryan's, but whose solution, even after the transient effects had died out, would vary aperiodically. I eventually found one which differed from Bryan's principally in that the thermal

forcing varied with longitude as well as latitude (Lorenz 1962a).
Upon establishing linear prediction formulas from the numerical output,
I found that indeed they did not yield perfect forecasts.

In the model the stream functions in the upper and lower layers
were denoted $\psi + \tau$ and $\psi - \tau$, and the potential temperatures were
denoted by $\theta + \sigma$ and $\theta - \sigma$, after which τ was identified with θ through
the geostrophic relation. In the final formulation ψ, τ, θ, and σ
were defined over an infinite strip bounded on the south and north
by the lines $y = 0$ and $y = \pi/\ell$, instead of over a sphere. The chosen
orthogonal functions, satisfying the appropriate boundary conditions
for ψ and τ, were

$$\phi_0 = 1 \quad , \tag{4.1}$$

$$\phi_1 = 2 \sin \ell y \cos k x \quad , \tag{4.2}$$

$$\phi_2 = 2 \sin \ell y \sin k x \quad , \tag{4.3}$$

$$\phi_3 = \sqrt{2} \cos \ell y \quad , \tag{4.4}$$

$$\phi_4 = 2 \sin 2\ell y \cos k x \quad , \tag{4.5}$$

$$\phi_5 = 2 \sin 2\ell y \sin k x \quad , \tag{4.6}$$

$$\phi_6 = \sqrt{2} \cos 2\ell y \quad ; \tag{4.7}$$

ψ was expressed in terms of $\phi_1, \ldots, \phi_6$, θ in terms of $\phi_0, \ldots, \phi_6$, and σ in terms of ϕ_0 alone. If σ had been allowed to vary horizontally, the equations would have been rather awkward to solve.

The LOM which had taken so long to develop proved to have many useful by-products. We mention first its application to the laboratory experiments of Fultz (1953), Hide (1953), and others. In brief, a rotating cylinder or annulus containing water is heated at the outer radius and cooled at the center or inner radius. With slow rotation, or with more rapid rotation and either very weak or very strong heating, the resulting flow is symmetric, i.e., independent of "longitude", but with more rapid rotation and moderate heating a set of waves develops. These may progress without changing their form, they may vacillate, i.e., vary their form periodically, or they may vary with an irregularity reminiscent of the atmosphere.

We had previously hypothesized (Lorenz 1953) that the wave regime would ensue when the symmetric flow became baroclinically unstable, and furthermore that the stability of the flow under strong heating, despite the large horizontal temperature gradient and accompanying vertical "wind" shear, resulted from the high static stability produced by the rapid large-scale overturning. The LOM appeared to be as good a model of the laboratory experiments as of the atmosphere, and, in view of its time-variable static stability, it offered a means of testing the hypothesis.

In addition, if all the variables and constants in the LOM with subscripts 4, 5, or 6 initially vanish, they continue to vanish, and may be discarded, so that the remaining system of eight equations

is also a complete LOM. It has by now been truncated to the point
where the waves can no longer effect a net cross-latitude transport
of momentum, but they can still transport sensible heat, so that the
model is still potentially useful for studying baroclinic instability.
The reduced model was easily modified to apply to a cylindrical region
instead of a channel; the only changes were in the numerical values
of some of the constants. It proved to be easy to solve analytically
for the steady state, and to determine its stability; the criterion
for baroclinic instability showed excellent qualitative agreement
with Fultz's experimentally determined transition curve. Further-
more, when four more variables were added to the model, so that waves
of two consecutive wave numbers were represented, analytic solu-
tions for the wave-number transitions closely duplicated the experi-
mental results (Lorenz 1962b).

To study the phenomenon of vacillation we returned to the
14-variable model (Lorenz 1963a). We were also forced to return to
numerical methods of solution. We found that, just beyond the limits
of baroclinic instability of the symmetric flow, steady waves would
develop (i.e., steady in a moving coordinate system), but, when these
limits were sufficiently far exceeded, vacillation would set in.
Moreover, the mechanism for vacillation was indicated as being the
barotropic instability of the flow which consisted of the symmetric
flow plus the superposed steady waves. Without the waves the symmetric
flow would have been barotropically stable; the role of baroclinic
instability in vacillation thus appeared to be the production of the waves.

In a final variation of the model (Lorenz 1965) we increased

the number of variables to 28, by allowing the simultaneous presence

of interacting waves of three different wave lengths, but suppressing

the variations of horizontally averaged temperature and static stability.

We then made a study of predictability, by first obtaining numerically

a "control solution" extending for 64 days, and then superposing

numerous small perturbations at various times during the 64 days. In

each case we determined how rapidly the perturbed solution would de-

part from the control run. This was the first of many systematic

studies indicating that small differences between solutions would

double in a matter of a few days (in this case, four days on the

average), and hence to imply that, while there might be considerable

room for improvement in one-week weather forecasts, accurate fore-

casting a month or more in advance was not possible.

Like conservative models, forced dissipative baroclinic models

soon found uses other than those originally planned for them   We

mention one application.

In investigating stochastic dynamic prediction and its relation

to predictability, Fleming (1971) introduced the concepts of certain

and uncertain energy. Sometimes all that we may know about a state

of a system is that it is a member of a particular ensemble. The

ensemble-mean energy, if it is quadratic, may be resolved into certain

and uncertain energy - the energy of the ensemble-mean state, and the

mean energy of the departures of individual states from the mean.

The latter affords a measure of uncertainty as to the precise state.

Both certain and uncertain energy may be resolved into kinetic and available potential energy. Fleming derived expressions for the generation and dissipation of these forms of energy, and the trans- formations among them.

As a case to which to apply these concepts, Fleming chose the 28-variable model mentioned above. He found that the major source of uncertain energy was certain available potential energy.

Thermally forced atmospheric circulations are not restricted to those where the heating contrast is horizontal. Small-scale convec- tive systems forced by heating from below or cooling from above have also been popular subjects for modelers. The forcing is ordinarily expressed in terms of a Rayleigh number.

One of the first convective LOM's was constructed by Saltzman (1962). Assuming uniformity in one horizontal direction, he expanded vertical cross sections of the stream-function and temperature fields in truncated double Fourier series, obtaining a seven-variable LOM, with three variables representing motion and four representing tem- perature. He obtained a number of numerical solutions where the initial state was a small departure from a state of steady convection. For larger Rayleigh numbers the system generally continued to oscillate. In one instance all but three of the variables decayed to zero, while those three continued to oscillate irregularly.

Having noted this aperiodic solution, we further truncated Saltzman's model by retaining only the three variables which did not decay, and confirmed the aperiodicity (Lorenz 1963b). Although too

highly truncated to provide a good representation of convection, the
new three-variable model has attracted much attention from mathe-
maticians as a simple system which varies aperiodically, despite its
deterministic formulation. Certainly no deterministic model with
fewer than three variables can undergo continual aperiodic oscillations.

Recently Sutera (1980) added small-amplitude random forcing to
the three-variable convective model. He obtained qualitatively
similar aperiodic behavior at Rayleigh numbers which would otherwise
have been subcritical.

Seeking a system which would undergo a still greater variety
of regime transitions, Shirer and Dutton (1979) constructed a model
of moist convection, in which they assumed that all upward motion was
saturated and all downward motion was unsaturated. They found that
six variables were sufficient to produce a wealth of regimes. Shirer
(1980) subsequently extended the study to an 11-variable model, in
order to include the effects of a prespecified wind shear in the
environment. For suitable forcing the model produced parallel cloud
bands, whose orientation was determined by the wind shear.

Even though the atmosphere is thermally forced, it is possible
to include forcing and damping in a barotropic model; the forcing
would have to be mechanical instead of thermal. One of the first
LOM's of this sort was formulated by Veronis (1963). The basic
equation was still the barotropic vorticity equation, but the model
represented flow in a square ocean basin, and the forcing was identi-
fied with wind stress. Veronis chose orthogonal functions of the form

sin mx sin ny, and truncated the series for ψ to four terms by letting m and n equal 1 or 2.

For weak forcing Veronis found a single stable steady solution, and in some instances two unstable steady solutions, but for stronger forcing there was sometimes, in addition to the stable steady solution, a stable but surprisingly complicated periodic solution. This was one of the first of many models found to possess two or more distinctly different "climates."

More recently, Charney and Devore (1979) developed a three-variable forced dissipative barotropic LOM, using the orthogonal functions ϕ_1, ϕ_2, ϕ_3 defined by (4.2)-(4.4). Their model differed from earlier ones in that they included mountains and valleys, with the form of ϕ_1. For suitable forcing they obtained two stable steady solutions, and were able to identify one of these solutions with the occurrence of blocking in the atmosphere. The study thus supported the hypothesis that geographical features play an essential role in the blocking phenomenon. Further support is afforded by a study by Charney and Straus (1980), who obtained similar results with 12-variable baroclinic LOM with mountains.

As a final example, we mention a 9-variable barotropic primitive-equation model with forcing, damping, and east-west mountain ridges (Lorenz 1980). The model illustrates the approach of an initially unbalanced state to quasi-geostrophic equilibrium. From this model a three-variable quasi-geostrophic LOM may be derived. A striking feature of the latter model is that although it represents barotropic

flow, it is identical to the three-variable convective model (Lorenz 1963b) described earlier. It therefore possesses aperiodic solutions for suitable values of forcing, damping, and mountain height. It thus reveals another property of LOM's which increases their potential usefulness; a LOM, having been constructed to represent one physical phenomenon, may prove applicable to a distinctly different one.

5. A model of a moist general circulation

We shall conclude our account with a description of a LOM which is still being developed, and has not yet been applied to specific problems. It is a model of the large-scale circulation of a moist atmosphere, and it includes thermodynamic and radiative effects of water vapor and liquid water. We present the model to illustrate the step-by-step construction of a LOM, but also to indicate how certain difficulties which have not arisen with previous LOM's may be handled, and to suggest possible future trends in low-order modeling.

For several reasons the incorporation of large-scale moist processes into a LOM is not straightforward. First, the nonlinear processes associated with water are not quadratic, and the computations are not easily performed with orthogonal functions. We shall handle this problem by using orthogonal-function coefficients as basic variables, and evaluating the spatial derivatives and advective terms as in dry models, but transforming at each time step to grid points, evaluating the remaining nonlinear terms at each grid point, and then transforming back to orthogonal functions. Obviously we must sacri-

fice some computational speed to do this.

Next, if orthogonal-function representations of temperature and water-vapor content are transformed to grid points, supersaturation may appear somewhere. We shall remove this possibility by using some measure of the total water content (vapor plus liquid) as a basic variable, and introducing an auxiliary formula to evaluate the water-vapor content at each grid point.

Finally, between the tropics and the polar regions the total-water mixing ratio may vary by one or two orders of magnitude. A highly truncated orthogonal-function representation might therefore transform to negative mixing ratios at high-latitude grid points. Accordingly, we shall use total dew point (the value which the ordinary dew point would assume if all the liquid water were vaporized) instead of total-water mixing ratio as the basic moisture variable; extreme values of total dew point should not differ by more than a factor of two.

In formulating the continuous equations from which the model will be derived, we shall choose pressure p as the vertical coordinate, and let the atmosphere be contained between the surfaces $p = 0$ and $p = p_0 = 1000$ mb; the height of the 1000-mb surface will be variable. The underlying surface will consist entirely of ocean. The system will be quasi-geostrophic.

Our basic dependent variables will be horizontal velocity expressed in terms of a stream function ψ and a velocity potential χ, individual pressure change ω, height z, temperature T, total dew point W, and sea-surface temperature S. Auxiliary variables will be the saturation

mixing ratios u, w, and s at pressure p and temperatures T, W, and S, and the water-vapor mixing ratio v; thus w will be the total-water mixing ratio.

The standard assumption for v is that it is the minimum of u and w. Since in a LOM a single grid point represents a large area, we prefer a formulation where a portion of the area may be subsaturated while another may contain clouds. A convenient formula which makes the liquid-water mixing ratio w-v small when the degree of subsaturation u-v is large, and vice versa, is

$$(u-v)(w-v) = \gamma^2 v^2 , \qquad (5.1)$$

where γ is a constant. Choosing $\gamma = 1/4$ makes the relative humidity $r = v/u = 0.8$ when $w = u$; the remaining water is in the form of clouds. We also note that choosing $\gamma = 0$ would reduce (5.1) to the standard assumption; thus, in a set of models with successively higher horizontal resolution, successively smaller values of γ might be appropriate.

We shall relate u and T by the formula

$$u = c' T^\mu p^{-1} , \qquad (5.2)$$

where c' is a constant and $\mu = L/R_w T^*$. Here L is the latent heat of condensation, assumed constant, R_w is the gas constant for water vapor, and $T^* = 273$ K is a typical atmospheric temperature. Analogous formulas will relate w and W, and s and S. Eq. (5.2) is a derivable

from an approximation to the Clausius–Clapeyron equation in which the factor $T*T$ replaces the factor T^2. An appropriate value for μ is about 20.0, and we shall choose $\mu = 20$ exactly, since much computation can be saved by evaluating integral instead of fractional powers. In fact, since in the radiation formulas we must compute T^4 in any case, we need only compute $(T^4)^5$.

Our basic diagnostic equations will be the thermal wind equation

$$\partial \psi / \partial p = -(R/f_0)\, T/p \quad , \tag{5.3}$$

obtained by eliminating z from the hydrostatic and geostrophic equations, where R is the gas constant for air and f_0 is the constant average value of the variable Coriolis parameter f, and the equation of continuity

$$\partial \omega / \partial p = -\nabla^2 \chi \tag{5.4}$$

Our prognostic equations will be

$$\partial \nabla^2 \psi / \partial t = -J(\psi, \nabla^2 \psi + f) - f_0 \nabla^2 \chi + \nabla^2 F, \tag{5.5}$$

$$d(c_p T + L w)/dt = RT\omega/p + H, \tag{5.6}$$

$$d w / dt = G, \tag{5.7}$$

$$d s / dt = E, \tag{5.8}$$

where c_p is the specific heat of air at constant pressure, F denotes the effects of friction, H denotes the atmospheric "heating", including the gain of latent heat through evaporation from the ocean, but not the effects of evaporation and condensation within the atmosphere, G denotes the effects of evaporation and precipitation, and E denotes oceanic heating. The simplified form (5.5) of the vorticity equation is consistent with the geostrophic approximation. By writing the thermodynamic equation (5.6) in terms of specific enthalpy $c_p T + Lv$, we include the thermodynamic effects of water. We note that dv/dt may be expressed in terms of du/dt and dw/dt through (5.1), and subsequently in terms of dT/dt, dW/dt and ω through (5.2).

We next reduce the vertically continuous equations to a form of the two-layer model in which we define ψ at two levels but T at only one. It is consistent with this formulation to define W at only one level. We shall obtain the new equations by integrating (5.6) and (5.7) vertically through the depth of the atmosphere, and (5.5) through the upper and also the lower 500 mb, or equivalently, through the upper 500 mb and the entire 1000 mb, instead of simply applying (5.6) and (5.7) at one level and (5.5) at two. The advantage of this procedure is that the vertical integrals $\overline{F}$, $\overline{G}$, and $\overline{H}$ of F, G, and H contain only the fluxes across the ocean-atmosphere surface and the top of the atmosphere; the integral $\widetilde{F}$ of F through the upper 500 mb also contains the momentum flux across 500 mb.

To perform the vertical integrations we must specify the vertical structure of the variables. We shall assume that in each vertical

column

$$T = T_0 \left(p/p_0 \right)^{\lambda} \quad , \tag{5.9}$$

where λ is a constant, whence, from (5.3) and (5.2),

$$\psi = \psi_0 - \left(R/f_0 \lambda \right) T_0 \left(p/p_0 \right)^{\lambda} \quad , \tag{5.10}$$

$$\mu = \mu_0 \left(p/p_0 \right)^{\mu \lambda - 1} \quad . \tag{5.11}$$

Noting that $\lambda = 0$ would imply an isothermal lapse rate, while $\lambda = R/c_p = 2/7$ would imply a dry-adiabatic lapse rate, we arbitrarily choose an intermediate value 0.175. We shall also let the relative humidity v/u be constant in each column, whence, from (5.1), w/u is constant, so that

$$w = w_0 \left(p/p_0 \right)^{\mu \lambda - 1} \quad , \tag{5.12}$$

$$W = W_0 \left(p/p_0 \right)^{\lambda} \quad . \tag{5.13}$$

Finally, we let

$$x = x_0 \left(2p/p_0 - 1 \right) \quad , \tag{5.14}$$

whence, from (5.4)

$$\omega = -\nabla^2 \chi_0 \left(p^2/p_0 - p \right) \ .$$

(5.15)

When (5.9) – (5.15) are substituted into (5.5) – (5.7) and the
vertical integrations are performed, we obtain the equations of the
model, in which the basic dependent variables are the surface values
ψ_0, T_0, χ_0, W_0, and S, and the forcing functions are $\overline{F}$, $\widetilde{F}$, $\overline{G}$, $\overline{H}$, and
E. We can eliminate $\partial T_0/\partial t$ from the thermodynamic and thermal-vorticity
equations, obtaining an ω-equation expressed in terms of χ_0 , which although
awkward, is tractable.

We shall make $\overline{F}$ and $\widetilde{F}$ proportional to ψ_0 and T_0, evaporation and
precipitation proportional to $s - v$ and $w- v$, and sensible heat flux
proportional to S – T. Radiation is not so simple. A highly sophisti-
cated treatment would be pointless in a model which has been so sim-
plified in other respects, so we shall represent some properties of
radiation crudely and others not at all.

We shall assume a fractional cloud cover dependent on relative
humidity; in our first experiments it equals r^4. Solar radiation which
strikes the clouds will be totally reflected; that which misses the
clouds will penetrate the atmosphere and heat the ocean. We shall
express solar radiation in terms of a "planetary temperature" T_Q.

We shall let the water-vapor spectrum possess a window through
which a fixed fraction a' of the radiation emitted by the ocean,
proportional to S^4, and not striking the clouds, passes to outer space.

The remaining fraction $1 - a'$ is absorbed by the atmosphere, as is all long-wave radiation striking the clouds. The atmosphere radiates upward and downward respectively at rates proportional to T_a^4 and T_b^4, where T_a and T_b are the temperatures of the uppermost and lowest 0.3 mm of precipitable water; the radiation from the cloud-free portion is diminished by the factor $1 - a'$.

To obtain a LOM, we shall express the fields of ψ_0, T_0, W_0, S, and T_Q in terms of the orthogonal functions $\phi_0, \ldots, \phi_6$, defined by (4.1) - (4.7), and used in the vacillation study. We may consequently anticipate vacillating and irregular as well as steady behavior. Since the constant term in ψ_0 is meaningless, there are 27 prognostic equations. As noted, at each time step we must transform back and forth from orthogonal functions to grid points.

Ideally the heat capacity of the ocean, or its mixed layer, should be large, but equilibrium states are approached more rapidly if it is set to zero. Eq. (5.15) then becomes the diagnostic equation $E = 0$, and the number of prognostic equations reduces to 20, making the output easier to diagnose.

Our preliminary experiments have yielded some interesting results. First, if T_Q is horizontally uniform, the system approaches a horizontally uniform state of rest, but, for some values of T_Q near 275 K, there are two equilibria—a very cold one and a very hot one. This situation evidently results from a cloud-albedo feedback process. There is little evidence that it is realistic; however, we do not really know how the atmosphere would behave if solar heating were uniform

over the globe.

Next, when we allowed T_Q to vary with latitude only, so as to produce a Hadley circulation, we first produced tropical temperatures above the boiling point of water and polar temperatures near absolute zero. We suspected an instability associated with the assumption of a fixed lapse rate, which would exceed the moist-adiabatic at high enough temperatures, but we finally found that the difficulty resulted from the choice of grid points, and that it disappeared when the number of grid points in the y-direction equalled the number of zonal orthogonal functions (ϕ_0, ϕ_3, and ϕ_6). We thus learned something about constructing LOM's with orthogonal-function to grid-point transformations.

Finally, when we perturbed an established Hadley circulation with a sufficiently strong cross-latitude temperature contrast, we obtained waves of reasonable amplitude progressing at a reasonable speed, and the associated moisture and vertical motion patterns were reasonable. We therefore believe that a low-order model of a moist general circulation is feasible.

6. Concluding remarks

Low-order models of the atmosphere, originally conceived as a means of illustrating some of the effects of nonlinearity, have proven useful in investigating specific problems, some of which could not readily be studied by other means. We have described a considerable variety of uses to which they may be put. We have seen that as com-

puters have become more powerful, and as tasks which might have been considered appropriate for LOM's have become suitable for larger models, still larger tasks have become suitable for LOM's.

Despite the speed with which extended numerical solutions of LOM's can be generated, we feel that there remain some tasks which will not be easily performed with LOM's until still faster computers are available. Among these, some may require extended runs with a more slowly operating LOM, such as the moist model which we have described. Others, which may not involve the higher-degree nonlinearity of the moist model, include the extension of a single run for a long enough time to permit climatic changes with the periods of ice ages, generation of very large ensembles of solutions to obtain fairly precise estimates of ensemble statistics, and such specific tasks as the determination of the normal modes of a system about a basic state which is not steady, but which is undergoing a complicated periodic cycle.

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SCIENTIFIC PERSONNEL

Principal Investigator: Edward N. Lorenz

Research Assistants: Wesley Ebisuzaki
Shun Der Ko
Thomas Nehrkorn